CURRENT DENSITY DISTRIBUTION AT THE PERIPHERY OF SUBMICRON PROBES

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1. Electrohydrodynamic ion sources are extremely attractive for forming submicron ion beams although the beams from them exhibit one unfavorable feature, i.e. the current density at a considerable distance r from the center of the beam falls quite slowly, approximately as $r^{-3.3}$ [Fig. 1, where curve 1 is the beam current density profile (nominal units) with residual gas pressure $p = 4 \cdot 10^{-5}$ Pa [1], 2 is the profile of beam gauss $\exp(-r^{-2}/0.32^2)$]. In ion lithography this leads to an effect similar to the proximity effect, i.e., with exposure of a resist neighboring regions are illuminated by "tails" of the beam and the picture is distorted [2].

Appearance of "tails" may be caused for the following reasons: dissipation of ions in residual gas, chromatic aberrations of the lens for ions from the "tails" of distribution functions with respect to energy, and mutual dissipation of ions leading to tails for distribution functions with respect to transverse pulses. It was detected in [1] that with a reduction in residual gas pressure to $p = 4 \cdot 10^{-5}$ Pa the current density in tails ceases to depend on pressure i.e., dissipation in the residual gas becomes immaterial.

Measurement of energy scattering tails made in [3] shows that the number of ions with high energy deficiency ΔE decreases approximately as $\exp(-\Delta E)$, i.e., tails of energy scatter cannot provide a slow stepwise reduction in current density. The reciprocal scattering of beam ions considered below was also worked out in [4] by the Monte Carlo method.

2. Each beam creates a natural electric field which may be broken down conditionally into two parts. The first is due to the total spatial charge and it causes orderly expansion of the beam, and the second causes discreteness of the charge and it fluctuates randomly in time and in space causing scattering of ions.

If the ions in a beam are arranged randomly the intensity of the fluctuating electric field obeys the Holtzmark distribution function [5]. With field evaporation of ions their arrangement around the emitting surface is not entirely random since an escaping ion with its own charge reduces the field at the surface impeding formation of the next ion. However, as a result of the fact that a beam expands rapidly and only in a direction perpendicular to its axis, ions are transported and their arrangement is chaotic.

The "tails" of a beam arise due to the presence of regions with a strong electric field ($F \gg en^{2/3}$, e is charge, n is ion density in the beam) and they may be calculated from the Holtzmark distribution although in this case it is difficult to consider transport of ions and dispersion of them due to mutual repulsion (these problems are also discussed in [6] where fluctuating fields $F \sim en^{2/3}$ are considered and an expression is obtained for the virtual size of a source). Since a strong field in the area of location of a "test" ion is created as a rule only by one "field" ion close by, then tails of the distribution function with respect to transverse velocities may be calculated by considering paired interaction and transport effects are considered automatically. Of course there are cases when alongside the test ion there are two or more field ions, although with large fields the relative significance of these cases becomes disappearingly small. Since accurate calculation of paired interaction is technically very complicated we use an approximation in which the trajectory of ions under the action of mutual repulsion is almost unchanged, and then we estimate the its applicability.

3. Shown in Fig. 2 are two closely placed ions at distance Z from the emitter found at point O. Ion A is placed at the center of a local coordinate system which moves together with it along axis Z, and the coordinates of ion B in this system are z, ρ. We shall consider that at instant of time $t = 0$, Z_0 is the coordinate of ion A (and z_0 is much less than the

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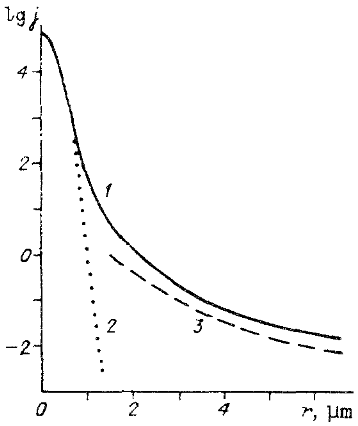

Fig. 1

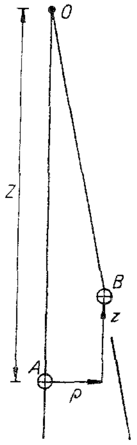

Fig. 2

length of drift interval L), Z_0 and ρ_0 are the coordinates of ion B. Since we ignore displacement of ions due to mutual repulsion then as there is movement $z = z_0$, $\rho = \rho_0 Z/Z_0$.

As shown in [6], the increase in distance of the tangent to the trajectory of an ion from the emitter in the plane of the emitter (in [6] this distance is called the virtual size)

$$dr = \frac{F_\perp}{2U} Z\,dZ. \tag{3.1}$$

Here U is voltage in the acceleration interval; $F_\perp$ is transverse fluctuating field operating on the ion:

$$F_\perp = \frac{e}{(\rho^2 + z^2)}\frac{\rho}{\sqrt{\rho^2 + z^2}}. \tag{3.2}$$

By substituting (3.2) in (3.1) and integrating from Z_0 to $L \gg Z_0$, we obtain

$$r = \frac{eZ_0^2}{2U\rho_0^2}\left(\ln \frac{L\left(1 + \sqrt{(z_0 Z_0/\rho_0 L)^2 + 1}\right)}{Z_0\left(1 + \sqrt{(z_0/\rho_0)^2 + 1}\right)} + \frac{1}{\sqrt{(z_0/\rho_0)^2 + 1}} - \frac{1}{\sqrt{(z_0 Z_0/\rho_0 L)^2 + 1}}\right). \tag{3.3}$$

By assuming that $z_0 \gg \rho_0$ (we demonstrate the correctness of this assumption below) and considering that $L \gg Z_0$, (3.3) is presented in the form

$$r = \frac{eZ_0^2}{2U\rho_0^2}\left(\ln \frac{L}{Z_0} - \ln \frac{z_0}{\rho_0}\right). \tag{3.4}$$

The probability that at instant of time t = 0 the coordinates of ion B will be ρ_0, z_0 is

$$dW = n2\pi\rho_0 d\rho_0 2dz_0 \tag{3.5}$$

(the two before z_0 appears due to the fact that we only consider $z \geq 0$). We express dz_0 from (3.4) as

$$dz_0 = -\frac{L}{Z_0}\frac{2U\rho_0^3}{eZ_0^2}\exp\left(-2Ur\rho_0^2/eZ_0^2\right) dr$$

and we place it in (3.5):

$$dW = 8\pi n \frac{LU}{eZ_0^3}\exp\left(-2Ur\rho_0^2/eZ_0^2\right)\rho_0^4\, d\rho_0\, dr. \tag{3.6}$$

By integrating (3.6) from $\rho_0 = 0$ to $\rho_{0\max} = \left(\frac{eZ_0^2}{2Ur}\ln\frac{L}{Z_0}\right)^{1/2}$, we find that

$$dW = \sqrt{2}\pi n Z_0^2 L\left(\frac{e}{U}\right)^{3/2}\frac{dr}{r^{5/2}}\left(\frac{3\sqrt{\pi}}{8}\operatorname{erf}(\sqrt{\lambda}) - (\lambda^{3/2}/2 + 3\lambda^{1/2}/4)\exp(-\lambda)\right) \tag{3.7}$$

[erf is probability integral, $\lambda = \ln(L/Z_0)$].

We estimate the value of λ when the energy of ions is constant (this corresponds to expansion of a beam from a crossover, whereas with expansion from a source there is acceleration of ions). The local coordinate system introduced above is only fit for the case when $z_{0\max} \le Z_0$, which determines the value of λ. In turn we find $z_{0\max}$ from (3.4). As a result of this

$$Z_{0\min} = z_{0\max} = L\frac{\rho_0}{Z_0} \leqslant L\left(\frac{e\lambda}{2Ur}\right)^{1/2} \quad \text{or} \quad \lambda \geqslant \ln\left(\frac{2Ur}{e\lambda}\right)^{1/2}.$$

With an energy eU = 10-50 keV typical for ion devices and for r = 1 μm we obtain $\lambda \ge 8$, whence the bracket in (3.7) may set as $3\sqrt{\pi}/8$. Since the probability integral erf(x) is set mainly with $x \le 1$, this relates to $\rho_0 \ll z_0$.

The value of nZ_0^2 in (3.7) is expressed in terms of angular beam intensity I and the energy of ions:

$$dW = \frac{3\pi^{3/2}}{8}\frac{ILm^{1/2}}{U^2}\frac{dr}{r^{5/2}} \tag{3.8}$$

(m is ion mass).

Expression (3.8) is the probability that in plane L two ions (since ions A and B are equal) appear to an observer to escape from certain points which are at distance r from the emitter. The apparent current from a ring of radius r

$$dJ = J_0 2dW$$

(J_0 is total beam current). The current density $j(r) = dJ/(2\pi r dr)$ or

$$j(r) = \frac{3\sqrt{\pi}}{8}J_0\frac{ILm^{1/2}}{U^2 r^{7/2}}. \tag{3.9}$$

It is interesting to compare the results for the two-particle approximation with the Holtzmark distribution. Using the asymptotic two-dimensional Holtzmark distribution [6]

$$dW(F_\rho) = \pi\frac{\Gamma(7/4)\Gamma(1/2)}{\Gamma(9/4)}ne^{3/2}\frac{dF_\rho}{F_\rho^{5/2}} \quad \text{for } F_\rho \to \infty$$

(Γ is gamma-function), and also Eq. (3.1), and comparing the results with (3.7), we find that for strong fields the mixing equals one [6].

4. In order to estimate the effect of beam tails with exposure of a picture we consider the following problems. We shall illuminate the whole plane by a narrow beam with current J_0 excluding a circle of radius R by illuminating each point for the same time τ as for exposure of an extended point. It is apparent that $\tau = \sigma_0/j_0$ (σ_0 is the required dose, j_0 is current density at the center of the beam). With illumination of ring of radius r and width dr the dose at the center of the ring

$$d\sigma = j(r)\tau\frac{2\pi r\,dr}{S}$$

(S is beam at the target, j(r) is current density distribution (3.9)). By substituting the required values and integrating from R to infinity, we find that over the center of the circle

$$\xi = \frac{\sigma}{\sigma_0} = \frac{\pi^{3/2}}{2}\frac{ILm^{1/2}}{U^2R^{3/2}}. \tag{4.1}$$

For a beam of Ga^+ the typical value I = 100 μA/rad² with U = 20 kV and L = 20 cm, and from (4.1) we have $\xi = 0.04$ with R = 1 μm, $\xi = 0.11$ with R = 0.5 μm, $\xi = 0.45$ with R = 0.2 μm. Thus, it is necessary to consider tails of an ion beam with exposure of a picture with element dimensions less than 0.5 μm.

5. If two ions are very close to each other, due to repulsion their trajectories are distorted. We estimate in which cases it is necessary to consider this effect.

We present a transverse fluctuating field in the form of the sum of two terms: $F_\perp = F_{\perp 0} + \delta F_\perp$. Here $F_{\perp 0}$ is an undisturbed field; $\delta F_\perp$ is the change for this field due to displacement of ions as a result of repulsion. To a first approximation

$$\delta F_\perp = \frac{\partial F_{\perp 0}}{\partial \rho}\delta\rho + \frac{\partial F_{\perp 0}}{\partial z}\delta z, \tag{5.1}$$

where $\delta\rho$, δz are deviation of an ion from an undisturbed trajectory, and $\delta\rho$ and δz are calculated assuming that an undisturbed field $F_{\perp 0}$, $F_{\parallel 0}$ operates on the ion. Repulsion of ions leads to a reduction in the electric field, and therefore $\delta F_\perp < 0$. Since expression (3.1) is linear with respect to $F_\perp$, the effect of additional field $\delta F_\perp$ may be considered separately. Deviation of an ion over the direction towards the emitter under the action of $\delta F_\perp$ (the derivation of the equation is given in the appendix, and we assume $\ln(z_0/\rho_0)$ is a slowly changing function) is presented as

$$\delta r = \frac{e^2 Z_0^4 \lambda k}{2U^2 \rho_0^4 z_0} \tag{5.2}$$

$(0 < k < 1)$. The probability that the initial coordinates of ion B would be ρ_0, z_0, is

$$dW_\delta = n 2\pi\rho_0 d\rho_0 2dz_0. \tag{5.3}$$

By means of (5.2) we express $d\rho_0$ in terms of $d(\delta r)$, and by integrating (5.3) with respect to z_0 we find that

$$dW_\delta = \pi\sqrt{2} n Z_0^2 \frac{e}{U} (\lambda k)^{1/2} z_{0\max}^{1/2} \frac{d(\delta r)}{\delta r^{3/2}}. \tag{5.4}$$

The value of $z_{0\max}$ is estimated with k = 0 or $z_{0\max} = \rho_0 L/Z_0$, whence

$$z_{0\max} = \left(\frac{e^2\lambda k L^4}{2U^2\delta r}\right)^{1/5}. \tag{5.5}$$

By substituting (5.5) in (5.4) we obtain the probability that two ions deviate at distance δr over the direction towards the emitter

$$dW_\delta = \pi 2^{2/5} n Z_0^2 \left(\frac{e}{U}\right)^{6/5} (\lambda k)^{3/5} L^{2/5} \frac{d(\delta r)}{(\delta r)^{8/5}}. \tag{5.6}$$

It is evident that expression (3.9) cannot be used when probability dW for deviation of an ion from the emitter at distance r becomes equal to the probability dW_δ for deviation of an ion towards the emitter at distance $\delta r = r$. Critical distance r is found be equating (3.7) and (5.6):

$$dW(r^*) = dW_\delta(r^*).$$

After simple calculations we have

$$r^* = \frac{(3\sqrt{\pi})^{10/9}}{2^{29/9}} \left(\frac{e}{U}\right)^{1/3} \left(\frac{L}{\lambda k}\right)^{2/3}.$$

Thus, with $r < r^*$ expression (3.9) remains correct and with $r > r^*$ current density falls more rapidly than $r^{-7/2}$. With typical parameters for a Ga beam (see above) and k = 1, $r^* =$ 2.4 μm, and if $k = \lambda^{-1}$ (and in fact the integral in (3.7) is selected with such k), then $r^* =$ 10 μm. As follows from part 4, the effect of beam tails on these distances is small, and distortion of the ion trajectories due to repulsion may be ignored.

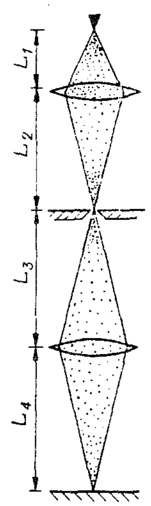

Fig. 3

6. Another reason which may lead deviation from (3.9) is the difference in longitudinal velocities due to finite energy scattering when two close ions fly past each other without managing to interact. If the distance between the ions is $\sim n^{-1/3}$, this effect is insignificant [6]. In the other limiting case when two ions are very close (which corresponds to large r in (3.4) it is possible to consider single-stage scattering by small angles. The angle at which ions are scattered is

$$\alpha = \frac{2e^2}{mv_{\parallel} v_0 \rho} = \frac{2e}{\delta U \rho}. \tag{6.1}$$

Here $v_{\parallel}$ is relative velocity; v_0 is total ion velocity; ρ is aiming parameter; $e\delta U$ is difference in energies. The probability that for time δt a field ion passes a test ion,

$$dW = 2\pi n \rho d\rho v_{\parallel} \delta t.$$

Considering that $\delta t = dZ/v_0$, $n = n_0 Z_0^2/Z^2$, $r = \alpha Z$, and taking account of (6.1) we obtain

$$dW = 4\pi n_0 Z_0^2 \frac{e^2 L}{\delta U U} \frac{dr}{r^3}. \tag{6.2}$$

Expression (6.2) gives the probability that the trajectories of two ions deviate by distance r. Here current density distribution

$$j(r) = 2\sqrt{2}\, J_0 \frac{IL\,(me)^{1/2}}{\delta U U^{3/2} r^4}. \tag{6.3}$$

We recall that (6.3) gives the distribution of current density with large r taking account of finite energy scattering. By equating (3.9) and (6.3) we find the critical distance

$$r_* = \frac{512}{9\pi} \frac{eU}{(\delta U)^2}.$$

With $r < r_*$ expression (3.9) is correct, but with $r > r_*$ it is (6.3). For typical Ga beam parameters ($e\delta U \sim 10$ eV) we have $r_* = 5$ µm.

7. We compare the results obtained with experimental data. Measurements are given in [1] for the dependence of ion probe tail intensity on residual gas pressure in a double-lens ion-optical column. Treatment of the results in [1] shows that with the minimum pressure ($p = 4 \cdot 10^{-5}$ Pa) the contribution of ion scattering in the gas is negligibly small. Our calculations are given for a single drift interval, although they are easily generalized for more complex cases (see the appendix). Double-charge ions Si^{++} and an accelerating voltage of 50 kV were used in [1]. The column is conditionally broken down into four parts (Fig. 3). According to the plan in [1] we estimate their length as $L_1 = 3.2$ cm, $L_2 = 6$ cm, $L_3 = 7.2$ cm, $L_4 = 7$ cm. We assume that the angular intensity in the last section $I = 10$ µA/rad^2 (the pro-

portion of Si^{++} in the alloy source is ~10% of the total current [7], but with acceleration the angular intensity increases). Results of calculations by Eq. (3.9) are presented in Fig. 1 by line 3. Since the beam diameter is large ($2r_0$ = 0.64 μm) tails are also noted at a considerable distance (r ≳ 2 μm). In narrower beams the tails should appear with smaller r. In our view the agreement with experimental data is satisfactory. It is necessary to remember that some data (I, L) are not accurately known and therefore rough values of them were used.

Thus, it is possible to conclude that beam tails arise due to the mutual scattering of ions. The harmful effect of tails may be felt in preparing pictures with element dimensions of ≤0.5 μm, and at these distances weakening of tails due to repulsion of ions and finite energy scattering is not significant. They may be suppressed to a considerable degree by working with a reduction in the last part of the column. It is evident that this effect may appear in autoelectronic emitters and in ion sources with field ionization.

Appendix. Deviation of an ion from an undisturbed trajectory is calculated by the equation

$$\delta\ddot{\rho} = eF_{\perp 0}/m, \quad \delta\ddot{z} = eF_{\parallel 0}/m. \tag{A.1}$$

By integrating Eq. (A.1) we obtain

$$\delta\rho = \frac{eZ_0^2}{2U\rho_0^2}\left[\frac{\Lambda-1}{\sqrt{\Theta^2+1}} - \ln\frac{\Lambda+\sqrt{\Theta^2+\Lambda^2}}{1+\sqrt{\Theta^2+1}}\right], \quad \delta z = \frac{eZ_0^2}{2U\rho_0^2}\left[\sqrt{1+\Lambda^2/\Theta^2} - \frac{\Theta+\Lambda/\Theta}{\sqrt{\Theta^2+1}}\right], \tag{A.2}$$

where $\Theta = z_0/\rho_0$; $\Lambda = Z/Z_0$. With integration we assume that $\rho = \rho_0\Lambda$. By substituting (A.2) in (5.1) and considering that of interest to us is the field of values $\Theta \gg 1$, $\Lambda \gg 1$, we have

$$\delta F_\perp = \frac{e^2Z_0^4}{2U\rho_0^5}\left[\frac{4\Theta^2\Lambda - 2\Lambda^3 + 3\Lambda^2}{\Theta(\Theta^2+\Lambda^2)^{5/2}} - \frac{(\Theta^2-2\Lambda^2)\ln\left(\Lambda/\Theta + \sqrt{1+\Lambda^2/\Theta^2}\right)}{(\Theta^2+\Lambda^2)^{5/2}} - \frac{3\Lambda}{(\Lambda^2+\Theta^2)^2}\right]. \tag{A.3}$$

We place (A.3) in (3.1) and integrate. We break down the integral into two parts: $1 < \Lambda < \Theta$ and $\Theta < \Lambda < L/Z_0$. The first integral is calculated assuming that $\Lambda \ll \Theta$, and the second that $\Lambda \gg \Theta$. Leaving only the main terms we obtain

$$\delta r = \frac{e^2Z_0^4}{2U^2\rho_0^4 z_0}\ln\frac{L}{Z_0\Theta} \quad \text{or} \quad \delta r = \frac{e^2Z_0^4}{2U^2\rho_0^4 z_0}\left(\lambda - \ln\frac{z_0}{\rho_0}\right).$$

We consider a double-lens ion-optical column similar to that used in [1] (Fig. 3). Let in the last section an ion deviate from the center of the probe by distance r_4. Since with identical ρ_0 the value of Z_0 is proportional to the length of interval L, from (3.4) the deviation of the trajectory for this ion from the crossover into the third section $r_3 = r_4(L_3/L_4)^2$, in the second $r_2 = r_3$, and in the first $r_1 = r_2(L_1/L_2)^2$. The total ion deviation at the target considering magnification will be

$$r = r_4 + r_3\frac{L_4}{L_3} + r_2\frac{L_4}{L_3} + r_1\frac{L_2}{L_1}\frac{L_4}{L_3},$$

$$r = r_4\left(1 + 2\frac{L_3}{L_4} + \frac{L_3}{L_4}\frac{L_1}{L_2}\right),$$

and the probability of deviation at distance r after the whole column

$$dW(r) = dw(r)\left(1 + 2\frac{L_3}{L_4} + \frac{L_3}{L_4}\frac{L_1}{L_2}\right)^{3/2},$$

where $dw(r) = \frac{3\pi^{3/2}}{2^{5/2}} nZ_0^2L_4\left(\frac{e}{U}\right)^{3/2}\frac{dr}{r^{5/2}}$; nZ_0^2 is expressed terms of I_4, i.e., the angular intensity in the last section. Current density in the plane of the target

$$j_t(r) = j_4(r)\left(1 + 2\frac{L_3}{L_4} + \frac{L_3}{L_4}\frac{L_1}{L_2}\right)^{3/2}$$

(j_4 is current density distribution due to expansion of the beam only in the last section). If there is a considerable reduction in the length of the last section, then due to the reduction in I_4 the value $j_4 \sim L_4^3$ and the current density in tails at the target falls as $L_4{}^{3/2}$.

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CALCULATION OF THE IRREGULAR INTERACTION OF SHOCK WAVES

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The problem is considered of irregular Mach interaction (reflection) of shock waves (SW). The flow structure is presented in Fig. 1a. At a certain point A there is formation of a reflected SW AB, a contact separation surface AL, and a strong SW AO with subsonic flow behind it. In the features of the mathematical definition of this problem it borders on the problem of spreading of a subsonic jet in an accompanying supersonic flow considered in [1]. In a nonviscous approximation Mach interaction of SW was considered in [2, 3] on the example of flow of an overexpanded jet in a flooded space. In the definition suggested in [1] it is possible to calculate Mach interaction taking account of gas viscosity.

1. In order to clarify the general features of Mach interaction we consider evolution of the interaction picture with an increase in the intensity of incident SW AA'. The intensity of this wave with a prescribed Mach number for the incident flow M_1, will be determined by the angle of its slope β_1. With regular interaction there is formation of a reflected SW whose slope angle is β_2 clearly depends on M_1 and β_1. With an increase in β_1 starting from some $\beta_1 = \beta_1{}^*$ two forms of interaction are theoretically possible: regular and Mach. For $\beta_1 > \beta_1^0$ only Mach interaction is possible; β_1^0 depends on M_1, or what is the same, on the ratio p_1/p_2, i.e., the pressures ahead of and behind an incident shock [4]. In spite of the fact that with $\beta_1 < \beta_2^0$ only regular reflection is observed by experiment, for the purposes of illustrating the effect of viscosity in pure form it is of interest theoretically with $\beta_1{}^* < \beta_1 < \beta_1^0$ to consider Mach interaction. With $\beta_1 > \beta_1{}^*$ the line of contact separation AL is directed towards the plane of symmetry, and the strong SW is curved. With departure from point A downwards over the flow along line AL the angle of slope of the contact surface tends towards zero and there is isoentropic compression of the outer supersonic flow. Mach number at line AL tends towards the value $M_3{}^*$ (β_1, M_1). With certain $\beta_1 = \beta_1{}^{**} > \beta_1^0$, $M_3{}^* = 1$, and the velocity equals sound velocity. For $\beta_1 < \beta_1^0$ flow in region ABL remains supersonic. The structure of flow in this case may be determined by only considering interaction between subsonic flow in jet OALO' and supersonic flow in region ABL. We call this form of irregular interaction isolated. With $\beta_1 > \beta_1^0$ only Mach interaction of SW is realized. For β_1 satisfying the condition $\beta_1^0 < \beta_2 < \beta_1{}^{**}$ flow along the whole of line AL with isoentropic compression remains supersonic. However, with interaction of compression waves formed in flowing around

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